RELATIVISTIC VIEW ON THE NUCLEAR EFFECTS IN THE DEEP INELASTIC SCATTERING ON DEUTRON

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Abstract

We analyze nuclear effects in the deep inelastic scattering on deuteron in the framework of the covariant approach. It is shown that this approach gives us the way to investigate the role of relativistic effects in the deep inelastic scattering (DIS) on deuteron, such as the relativistic kinematics and the off-mass-shell behavior of the Compton amplitude of nucleon in the consistent manner. We have obtained that taking into account of the nucleon amplitude off-mass-shell behavior gives us an analog of the interaction corrections in nonrelativistic models.

1 Introduction

The cross section for the deep inelastic scattering is proportional to the lepton and hadron tensors in the one photon approximation:

$$\sigma \propto L_{\mu\nu}(p,q)W^{\mu\nu}(P,q). \tag{1}$$

Where P, p and q are the initial momenta of the target, lepton and virtual photon, respectively. All information about target and its nuclear properties is concentrated in the hadron tensor which can be expressed in terms of the scalar structure functions:

$$W_{\mu\nu}(P,q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x) + \frac{1}{P\cdot q}\left(P_\mu - \frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}q_\nu\right) F_2(x). \tag{2}$$

In the Bjorken limit we can use the following relation:

$$\lim_{Q^2\to\infty} g_{\mu\nu}W^{\mu\nu}(P,q) = -\frac{1}{x}F_2(x).$$

Nuclear effects in the deep inelastic scattering appear as a difference of the structure functions of the bound and free nucleons. This can be provided, for example, by kinematics of bounded nucleons, off-mass-shell effects, and by existence inside deuteron of some additional degrees of freedom such as nucleon-antinucleon pairs, exchange mesons, Δ isobar, and others. Starting from the covariant approach [2] based on the Bethe-Salpeter formalism [1], we can analyze the role of relativistic kinematics and off-mass-shell effects in a consistent manner.

This approach allows us to write the impulse approximation for the hadron tensor in the following form:

$$W^D_{\mu\nu}(P,q) = \int \frac{d^4k}{(2\pi)^4} W^N_{\mu\nu}\left(\frac{P}{2}+k,q\right) f^N(P,k) + \int \frac{d^4k}{(2\pi)^4} W^{\overline{N}}_{\mu\nu}\left(\frac{P}{2}+k,q\right) f^{\overline{N}}(P,k). \tag{3}$$

Where k is the relative momentum of nucleons inside deuteron. The distribution functions $f^N(P,k)$ are expressed via the BS-vertex function $\Gamma(P,k)$:

$$f^N(P,k) = -\overline{\Gamma}_{\alpha\beta}(P,k) S^+_{\alpha\gamma}\left(\frac{P}{2}+k\right) S_{\beta\delta}\left(\frac{P}{2}-k\right)\Gamma_{\gamma\delta}(P,k)\frac{1}{2E\left(\left(\frac{P}{2}+k\right)_0 - E\right)},$$

$$f^{\overline{N}}(P,k) = -\overline{\Gamma}_{\alpha\beta}(P,k) S^-_{\alpha\gamma}\left(\frac{P}{2}+k\right) S_{\beta\delta}\left(\frac{P}{2}-k\right)\Gamma_{\gamma\delta}(P,k)\frac{1}{2E\left(\left(\frac{P}{2}+k\right)_0 + E\right)},$$

$$E = \sqrt{\left(\frac{\mathbf{P}}{\mathbf{2}}+\mathbf{k}\right)^2 + m^2}$$

Where $S(P,k)$ is a nucleon propagator and $S^{+(-)}(P,k)$ is the propagator of a nucleon with positive (negative) energy. The letters $\alpha, \beta, \delta and \gamma$ denote the spinor indices.

The elementary nucleon (antinucleon) amplitude $W^{N(\overline{N})}_{\mu\nu}\left(\frac{P}{2}+k, q\right)$ depends on the time component of the nucleon relative momentum, and square of momentum of this nucleon is not equal to its mass m. It leads to two difficulties at least. The first consists in changes of the representation (2) for this hadron tensor and the second is that we cannot use the structure function of a physical nucleon to calculate a deuteron one. To solve this situation, we have to express a deuteron hadron tensor in terms of physical nucleon tensors.

2 Expansion near Mass-Shell

As one can see, expression (3) is a convolution of the nucleon hadron tensor and distribution function which contains information about nuclear properties of the target. Inasmuch as deuteron is a weakly bounded system we can try to expand this function near mass-shell of the nucleon interacting with photon. To do it, one can rewrite nucleon propagator near the pole as

$$\frac{1}{\left(\left(\frac{P}{2}+k\right)_0 - E\right)^2} \simeq i\pi\delta'\left(\left(\frac{P}{2}+k\right)_0 - E\right). \tag{4}$$

Substitution of this relation in the expression for the deuteron hadron tensor gives us expansion of the latter in terms of the nucleon hadron tensor and its derivatives at $\left(\frac{P}{2}+k\right)_0 = E$. First, we can use the representation (2) to obtain the structure function. But the representation of a derivative of the on-mass-shell tensor consists of both the structure functions F_1, F_2 and the derivatives of additional structure functions. Neglecting derivatives of these additional structure functions, we can obtain the following expression for the deuteron structure function in the laboratory system (momentum of deuteron is $P = (M_D, \mathbf{0})$, momentum of photon is $q \simeq (q_0, 0, 0, q_0)$):

$$F_2^D(x_D) = \int \frac{d^3k}{(2\pi)^3}\frac{m^2}{4E^3(M_D-2E)^2}\left\{F_2^N(x_N)\left(1-\frac{E+k_3}{M_D}\right)\Phi^2(M_D,k) - \right. \tag{5}$$

$$\left.\frac{M_D-2E}{M_D}x_N\frac{dF_2^N(x_N)}{dx_N}\Phi^2(M_D,k) + F_2^N(x_N)\frac{E-k_3}{M_D}(M_D-2E)\frac{\partial}{\partial k_0}\Phi^2(M_D,k)\right\}_{k_0=E-\frac{M_D}{2}}$$

Here $x_D = \dfrac{-q^2}{M_D q_0}$ and $x_N = \dfrac{-q^2}{(E-k_3)q_0}$ are Bjorken variables for the nucleon and deuteron, respectively. The function $\Phi^2(M_D,k)$ is connected with the Bethe-Salpeter vertex function $\Gamma(P,k)$ in the rest frame of the deuteron:

$$\Phi^2(M_D,k) = \overline{\Gamma}_{\alpha\beta}(M_D,k)\sum_s u^s_\alpha(\mathbf{k})\overline{u}^s_\delta(\mathbf{k})\sum_{s'} u^{s'}_\beta(-\mathbf{k})\overline{u}^{s'}_\gamma(-\mathbf{k})\Gamma_{\delta\gamma}(M_D,k) + \overline{\Gamma}_{\alpha\beta}(M_D,k)\sum_s v^s_\alpha(-\mathbf{k})\overline{v}^s_\delta(-\mathbf{k})\sum_{s'} v^{s'}_\beta(\mathbf{k})\overline{v}^{s'}_\gamma(\mathbf{k})\Gamma_{\delta\gamma}(M_D,k). \quad (6)$$

Here we neglect terms with power of $\dfrac{<V>}{M_D}$ large than two; $<V>$ is the mean value of the nucleon-nucleon potential.

3 Nonrelativistic Limit

To compare our result with the nonrelativistic calculations (for example [3]), we expand E in powers $\dfrac{\mathbf{p}^2}{m^2}$ in (5) and discard the last term because of its pure relativistic behavior. That gives us the following expression for the structure function F_2^D:

$$\frac{1}{2}F_2^D(x_D) = \int \frac{d^3k}{(2\pi)^3}\left\{F_2^N(x_N)\left(1-\frac{k_3}{m}+\frac{\epsilon}{m}\right)\Psi^2(\mathbf{k}) - \frac{-T+\epsilon}{m}x_N\frac{dF_2^N(x_N)}{dx_N}\Psi^2(\mathbf{k})\right\} \quad (7)$$

$T = 2E - 2m$ is the kinetic energy of nucleons, $\epsilon = M - 2m$ is the binding energy. Here we introduce an analog of the nonrelativistic wave function $\Psi^2(\mathbf{k})$:

$$\Psi^2(\mathbf{k}) = \frac{m^2}{4E^2 M_D (M_D-2E)^2}\left\{\Phi^2(M_D,k)\right\}_{k_0 = E - \frac{M_D}{2}}$$

with the usual normalization condition:

$$\int \frac{d^3k}{(2\pi)^3}\Psi^2(\mathbf{k}) = 1.$$

Comparing (7) with nonrelativistic calculations we can conclude that we have got here an analog of the nonrelativistic impulse approximation with interaction corrections.

4 Conclusion

In the present talk we analyzed the relativistic impulse approximation for the deuteron structure function F_2^D. We droped out here some of the off-mass-shell effects connected with an additional structure functions in (3). This approximation requires additional investigation. Also, we have not taken into account the contribution from P-states to the Bethe-Salpeter vertex function. We will analyze its contribution in the near future. We have obtained the contribution of effects resulting from the Bethe-Salpeter vertex function dependence of relative nucleon energy (5). Comparing the obtained results with calculation in the nonrelativistic field theory model [3] allows us to conclude that the relativistic impulse approximation in the Bethe-Salpeter formalism contains both the effects of the Fermi motion and some mesonic effects. So relativistic calculation can shed light on nature of mesonic corrections in the nonrelativistic models.

References

[1] E.E. Salpeter and H.A. Bethe, Phys. Rev. **76** 769 (1949).

[2] V.V.Burov and A.V.Molochkov, Preprint JINR No. E2-95-339.

[3] L. Kaptari, K. Kazakov, A. Umnikov, Phys. Lett. **B293** 219-225 (1992).